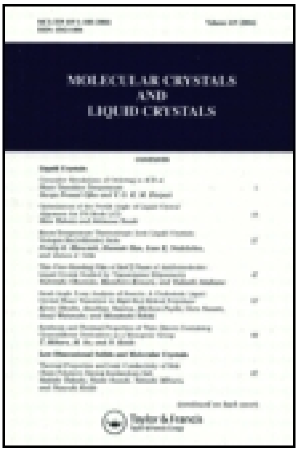

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A Mechanical Model for Phase Transitions in Smectics

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A mechanical model is a simple device made out of helical springs. Using the mechanical model one can visualize the difference between the first and the second order phase transitions. The analogy between the mechanical model and phase transitions in liquid crystals will be discussed. The parameters of the simple device (length, frequency, amplitude, tilt, etc.) can serve as an analogy for the parameters used in theories of phase transitions. The simple mechanical model is also convenient for the presentation and visualization of the dynamics near SmA-SmC phase transition in liquid crystals.

Keywords First order phase transition; mechanical model; second order phase transition; teaching liquid crystals

1. Introduction

There is an absence of topics based on modern materials in the curriculum of physics courses in Slovenia and probably in many other countries in the world. As the liquid crystals are one of these materials as well, there is only informal knowledge presented to students. There is also practically no transfer of knowledge between the educated researchers and teachers towards the students. As researchers we are responsible to enable the transfer of knowledge about recent developments and results of the researches into schools and to population in general. It is worth the effort to discuss and introduce these topics in a way that students will be able to understand basic concepts.

With the ambition to make a small step forward in the teaching process of modern materials and liquid crystals, we introduced the teaching unit on liquid crystals [1,2]. The proposed teaching approach includes mostly the constructivistic aspects. The first step is the introduction and manipulation with the liquid crystalline substance. The teaching unit includes all steps from the synthesis of a liquid crystal to preparation of the parallel/wedge cells and observation of the physical properties.

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Macroscopic phenomena are usually evident and persuasive, which was confirmed by the statements of students during and after the lab work.

Microscopic issues are more abstract and difficult to comprehend. Therefore models are often used. One of the models that is convincing due to the material it uses and due to the set up is a model for anisotropy. The anisotropy in liquid crystals finds the analogy in the anisotropy of wood [3]. Mechanical models are due to everyday experiences in mechanics, even more welcome in teaching.

The model presented in this contribution was developed with the goal to help visualize and construct the concept of phase transitions in liquid crystals, particularly the SmA-SmC phase transition. The model uses springs, which are analogues for molecules of liquid crystals. The shape of the spring is reminiscent to the shape of a liquid crystalline molecule. One of the order parameters in the SmC phase is the tilt that can be modeled directly by the tilt of springs' ends. It means that also in the model the tilt has similar meaning and appearance. Both types of phase transitions, the first and the second order, are usually found in materials having SmA-SmC phase transition. Both can be nicely presented with the mechanical model. The model also demonstrates dynamics and critical slowing down near phase transition.

2. Theory of the SmA-Smc Phase Transition

The order of tilted smectics is usually described by the two-dimensional order parameter $\vec{\xi}$ which gives the magnitude of the average tilt of elongated molecules in the layer and its direction. The free energy in the simplest form which allows for description of the second and the first order transition is given by

$$G = \frac{1}{2}a_0 \vec{\xi}^2 + \frac{1}{4}b_0 \vec{\xi}^4 + \frac{1}{6}c_0 \vec{\xi}^6 \tag{1}$$

Here only the first coefficient is temperature dependent $a_0 = a(T - T_c)$, where a is the material parameter and T_c is the temperature where the transition to the tilted phase takes place. The second coefficient b_0 is positive for the second order transition, while materials with the first order transition are described by negative b_0. The last coefficient c_0 is always positive.

Using the simple *ansatz* $\vec{\xi} = \theta(\cos\varphi, \sin\varphi)$ which takes into account the rotational symmetry of the system gives the temperature dependence of the tilt magnitude θ for the systems with second order phase transition (where usually c_0 is neglected)

$$\theta^2 = 0 \text{ for } T > T_c \text{ and } \quad \theta^2 = -\frac{a_0}{b_0} \text{ for } T < T_c \tag{2}$$

For the system having the first order transition, the temperature dependence of the tilt θ is:

$$\theta^2 = 0 \text{ for } T > T_c \text{ and } \theta^2 = \frac{-b_0 + \sqrt{b_0^2 - 4a_0c_0}}{2c_0} \text{ for T} < \frac{b_0}{4\,a\,c_0} + T_c \tag{3}$$

If temperature is found between T_c and $\frac{b_0}{4ac_0}+T_c$ than both solutions are stable and the phenomenon can be observed as a hysteresis with respect to the tilt on heating and cooling.

If the materials possess the second order phase transition also dynamical properties can be modeled. When approaching the transition temperature from above, the relaxation frequency of the soft mode decreases proportionally to $(T-T_c)$. Below the transition temperature, the mode splits in to two modes: the soft mode and the Goldstone mode. The soft mode increases upon lowering the temperature linearly having twice as steep slope of the frequency temperature dependence as the soft mode above the transition [4]. The Goldstone mode has the frequency zero consistent with the rotational symmetry of the structure. All described phenomena can be well presented by the analogy using the mechanical model.

3. Mechanical Model

A set up of a mechanical model is rather simple. For modeling the first order transition one needs a tension spring and for modeling the second order transition the compression spring. A metal rod that fits into springs, weights (plasticine) and a stand are also needed for the construction of the model (Fig. 1). A tension spring is made of close winds with initial tension designed in a way that it absorbs and store energy by offering resistance to a pulling force. A compression spring is a helical spring wound in open coils and is used to resist applied compression forces (Fig. 1).

The first order phase transition is modeled in the following way. The starting point for introduction of terms and parallels in smectics are the case of good interpretation of what students know from daily life phenomena (like water melting) and the phenomena that take place in the liquid crystals. The main issue in the frames of the model is to determine what the phase transition, when considering the model, is. The next step is to interpret the conditions and variables which are the analogues to the order parameters. By determination of the parallels between

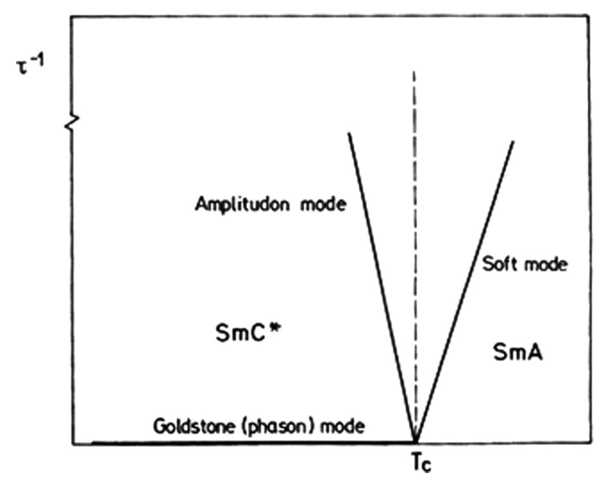

Figure 1. A set of equipment for the model: 1-compression spring (k = 63 N/m, l = 0,3 m), 2-tension spring (k = 74 N/m, l = 0,3 m), 3-metal rod, 4-stand, 5-plasticine weights.

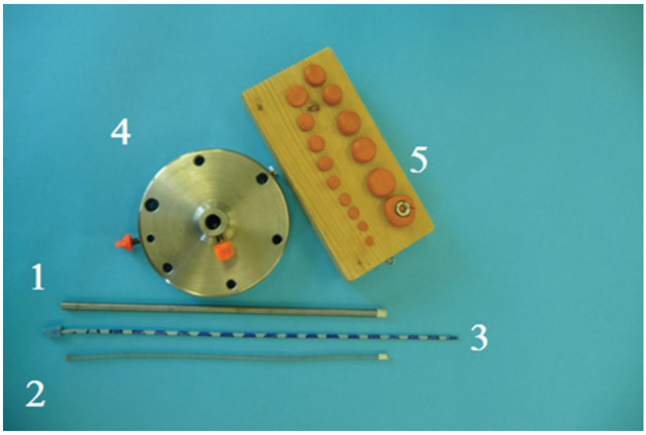

Figure 2. A spring that is straight (left) and turned over (right) i.e., before and after the phase transition. (Figure appears in color online.)

the model and the real system there can be done even more, the general influences and interdependences can be detected.

In the model of the first order phase transition the spring imitates the molecules. The tilt of the molecule is similar to the tilt of the spring's end, which both again looks alike. The phase transition is achieved when the spring tilts in such a manner that the spring overturns (Fig. 2).

The occurrence of the phase transition is the consequence of the changing conditions. In liquid crystals there is a change in temperature or the change in electric field; in the model there is a change in length of a spring or a change in the load of the spring (Table 1). From this one can make inferences that the change in length/mass of the load is an analog for the temperature changes in liquid crystals.

It is well known that the first order phase transition is described by the characteristic jump in the order parameter (Fig. 3) that needs to be demonstrated as well with the mechanical model. It can be nicely shown that the jump in the tilt of the spring suddenly occurs, when the length or load is large enough. The jump and the hysteresis can be shown by the tension spring only (Fig. 3).

The second order phase transition is another phenomenon that can be demonstrated in a same way just by using another type of spring – the compression spring. The continuous change of order parameter can be detected by changing the load mass or the length of a spring (Fig. 4). The load is changed by applying plasticine

Table 1. An overview of analogies between liquid crystals and mechanical model

Liquid crystals	Mechanical model
Phase transition	Overturn of a spring
1st order phase transition	Tension spring
2nd order phase transition	Compression spring
Change in temperature	Change in the length/load of the spring
Tilt	Tilt of a spring's free end

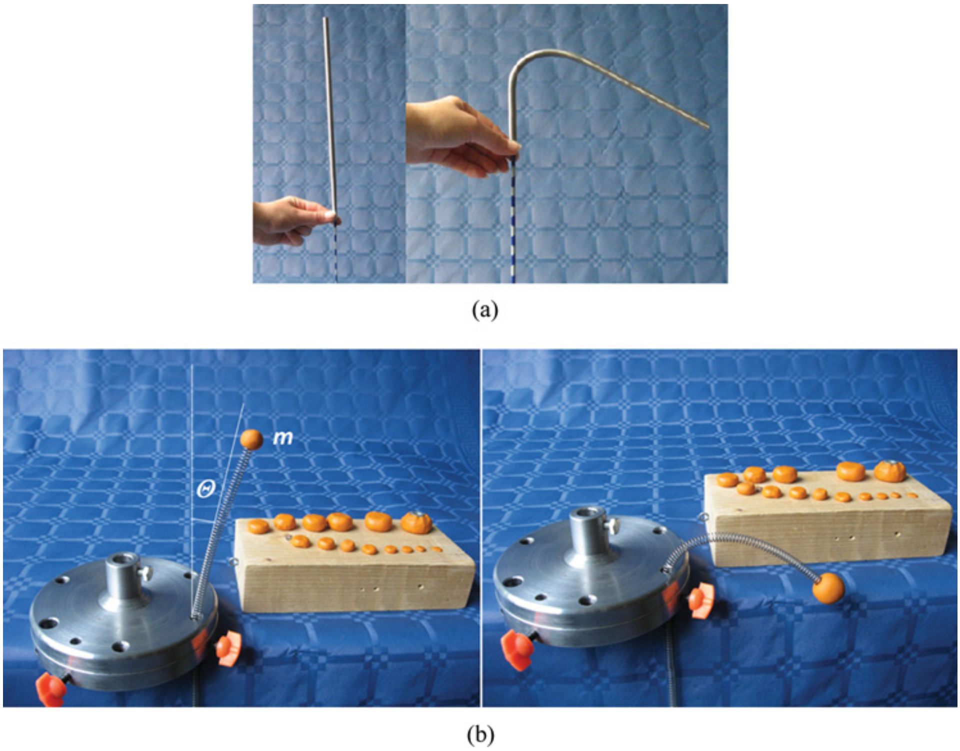

Figure 3. An analogy between the mechanical model and liquid crystals. When increasing the length of a spring the spring stays unbent until the critical length when the spring suddenly bends. The reverse procedure reveals that the spring remains in the bent state at lengths where it was straight before. In a certain length interval one can observe two different states (left). A characteristic jump in the order parameter at first order phase transition in liquid crystals (right). (Figure appears in color online.)

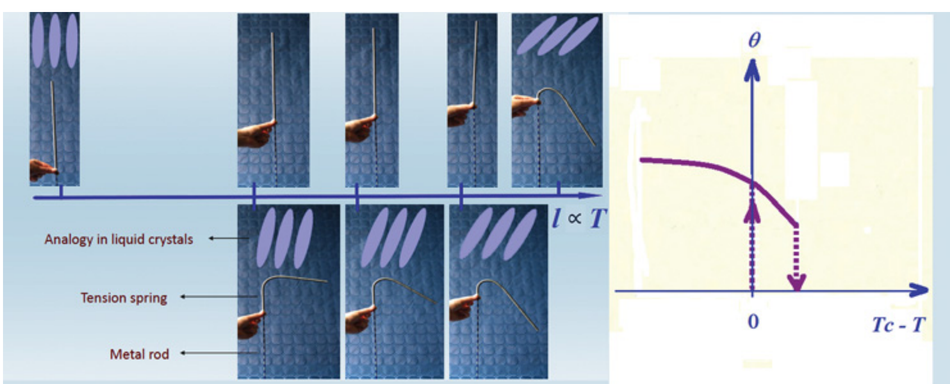

Figure 4. Change of the inclination with the length/load mass of a spring shows similar characteristic as in liquid crystals. The characteristic is represented in the graph of continuous change of order parameter (tilt) with the temperature in liquid crystals. (Figure appears in color online.)

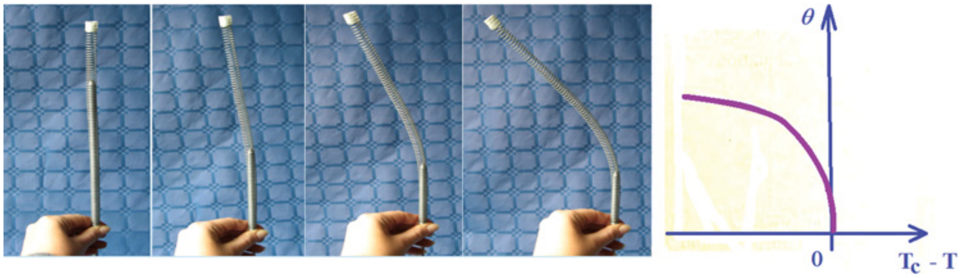

Figure 5. Splitting mode in the vicinity of the critical temperature [4]. (Figure appears in color online.)

balls of different masses on the free end of a spring. The change of the length can be assured in two ways: fixed on a stand or unfixed on a metal rod. When one fixes the spring in a stand the discrete length changes can be done, while the continuous changes in length can be achieved by metal rod. The rod is inserted into the spring in a way that it supports the spring; changes in the length of a rod inserted in the spring consequently exhibits in changes of the length of a spring (Fig. 3, Fig. 4).

In the vicinity of a second order phase transition the measurements of the dynamic properties of the model can be performed as well. The parallels between the model and in the liquid crystals can be found in the splitting modes (Fig. 5) near the SmA-SmC phase transition. When studying the dynamics of liquid crystals near phase transitions there are two characteristic modes that are accompanying the second order phase transition. The soft mode critically slows down when approaching the phase transition from above. At phase transition the soft mode splits into phase and amplitude modes below the SmA-SmC transition.

The measurements of the frequencies in dependence on the mass of the load near phase transitions can be performed by the optical gate device. The data analysis gives the average oscillating time for the certain length/load of the spring. The dependence between the oscillation time and the load of the free end of the spring is presented on the Figure 6. The critical slowing down of the oscillation frequency is clearly seen.

The frequency decrease is observed with increasing load. The shape of the graph (Fig. 6) is not completely similar to the theoretical graph for liquid crystals (Fig. 5)

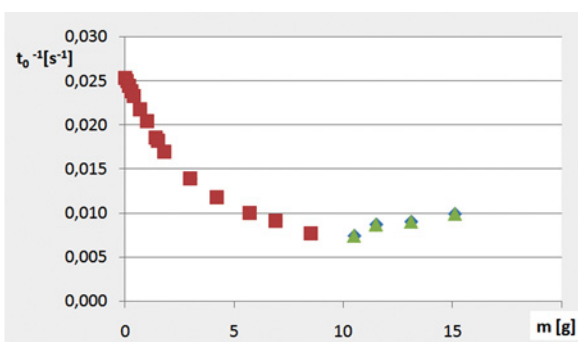

Figure 6. Measurements of the oscillation frequency in dependence on the load fixed on the free end of a spring. (Figure appears in color online.)

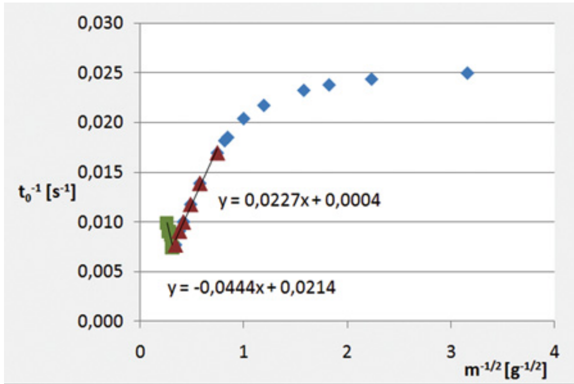

Figure 7. The graph shows the agreement with the theoretical soft mode (red triangles) and the amplitudon mode (green squares) near the point of the critical load. The slopes of the two series differ for the factor of 2. Blue diamonds indicate the region far from the phase transition. (Figure appears in color online.)

however one has to bear in mind, that the frequency has to be compared with the $1/\sqrt{m}$, as the magnitude of the tilt in liquid crystals is proportional to $\sqrt{(T_c - T)}$ (Fig. 7).

When observing the graph near the critical load, one can see the linearity of the slopes that are in good agreement with the theory of splitting modes near the SmA-SmC phase transition. The linearity disappears in a region far from the phase transition where the load of a spring is small (blue diamonds in the Fig. 8). The two linear series coincide with the soft mode and the amplitudon mode in liquid crystals (Fig. 5, Fig. 7). The missing Goldstone mode is not plotted on the graph but can be also modeled by the same model. The Goldstone mode in the mechanical model is visualized by the set up where the spring is not rigidly fixed in on the stand, but is fixed in a way that it can rotate freely. The possibility for overturn of the spring in the space is thus the same for any chosen direction in the space. One can observe the oscillation of the spring (amplitudon mode) in any direction in the plane perpendicular to the rotation axis (Goldstone mode) i.e. the spring can freely rotate in the space during the oscillation of the free end of the spring. The same mechanical model with a small modification in the spring fixation therefore assures the visualization of the splitting modes.

4. Conclusions and Perspectives

The mechanical model serves as a universal model for describing and visualizing both types of the phase transition. However the analogy is the most straightforward for liquid crystals. A simple set up is convenient since it needs only widely accessible materials, which are present in any school lab. The visualization of the dynamics and accompanying measurements are useful during the process of learning and construction of the concepts in the theory of liquid crystals as well. The applicability of the model is wide-ranging (from chemistry to physical courses; from primary level up to the undergraduate and graduate level). The efficiency of the model will be reported after the testing among university students on the physics courses for future (physics) teachers.

References

[1] Pavlin, J., Susman, K., Vaupotič, N., & Čepič, M. (2010). How to teach liquid crystals. In:. *23rd International Liquid Crystal Conference*, 11–16 July, Krakow, Poland. *Abstract book.*, *256*.
[2] Ziherl, S., Susman, K., Čepič, M., Pavlin, J., Glažar, S. A., & Vaupotič, N. (2010). *XIV. IOSTE Symposium, International Organization for Science, and Technology Education, June 13.-18. 2010, Bled, Slovenia. Socio-cultural, and human values in science, and technology education: proceedings.* Dolinšek, S. (Editor). 1-7, Institute for Innovation and Development of University: Ljubljana.
[3] Ziherl, S., Bajc, J., Urankar, B., & Čepič, M. (2010). *Eur. J. Phys.*, *31*, 531–542.
[4] Muševič, I., Blinc, R., & Žekš, B. (2000). *The physics of ferroelectric and antiferroelectric liquid crystals*, World Scientific, cop.: Singapore.